

Universal R - C crossover in current-voltage characteristics for unshunted array of overdamped $Nb - AlO_x - Nb$ Josephson junctions

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We report on some unusual behavior of the measured current-voltage characteristics (CVC) in artificially prepared two-dimensional unshunted array of overdamped $Nb - AlO_x - Nb$ Josephson junctions. The obtained nonlinear CVC are found to exhibit a pronounced (and practically temperature independent) crossover at some current $I_{cr} = \left(\frac{1}{2\beta_C} - 1\right) I_C$ from a resistance R dominated state with $V_R = R\sqrt{I^2 - I_C^2}$ below I_{cr} to a capacitance C dominated state with $V_C = \sqrt{\frac{\hbar}{4eC}}\sqrt{I - I_C}$ above I_{cr} . The origin of the observed behavior is discussed within a single-plaquette approximation assuming the conventional RSJ model with a finite capacitance and the Ambegaokar-Baratoff relation for the critical current of the single junction.

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Among many different properties which can be studied using highly ordered two-dimensional arrays of Josephson junctions probably one of the most interesting (and important for their potential applications) is their transport behavior, reflecting manifestation of numerous dissipation mechanisms in the arrays (see, e.g.,¹⁻³ and further references therein) via nonlinear current-voltage characteristics (CVC) of the general form $V \propto [I - I_C(T)]^{a(T)}$ with a power exponent a ranging from $a < 1$ to $a > 1$ depending on the particular mechanism and collateral effects (such as finite size of the single junction and/or the array, thermal fluctuations, quasi-particle contributions, etc⁴⁻⁹). At the same time, recall that only sufficiently overdamped Josephson junctions (with nonhysteretic CVC) and their arrays can be effectively used in rapid single flux quantum (RSFQ) logic circuits and programmable Josephson voltage standards (see, e.g.,¹⁰⁻¹³ and further references therein). In this paper we report our results on CVC for SIS type array of strongly overdamped

$Nb - AlO_x - Nb$ junctions at different temperatures. We observed quite a pronounced crossover at some current $I_{cr} = \left(\frac{1}{2\beta_C} - 1\right) I_C$ between a resistance R dominated state (below I_{cr}) and a capacitance C dominated state (above I_{cr}) which could be utilized as a versatile R - C switch within Josephson electronics. High quality ordered SIS type unshunted array of overdamped $Nb - AlO_x - Nb$ junctions has been prepared by using a standard photolithography and sputtering technique¹. It is formed by loops of niobium islands linked through 100×150 tunnel junctions. The unit cell of the array has square geometry with lattice spacing $a \simeq 46\mu m$ and a single junction area of $5 \times 5\mu m^2$. The critical current for the junctions forming the arrays is $I_C(T) = 150\mu A$ at $T = 1.7K$. Given the values of the junction quasi-particle resistance $R = 10\Omega$ and capacitance $C = 1.2fF$, the circuit frequency and dissipation measuring Stewart-McCumber parameter are estimated to be $\omega_{RC} = 1/CR \simeq 10^{14}Hz$ and $\beta_C(T) = \frac{2\pi CR^2 I_C(T)}{\Phi_0} \simeq 0.05$ at $T = 1.7K$, respectively. The parameters of the array are as follows, $I_{CA}(1.7K) = 1.2mA$ and $R_A = 25\Omega$. The measurements were made using home-made experimental technique with a high-precision nanovoltmeter⁸. The temperature dependence of the normalized critical current of the array $I_{CA}(T)/I_{CA}(0)$ is shown in Fig.1. Observe that it is very well fit-

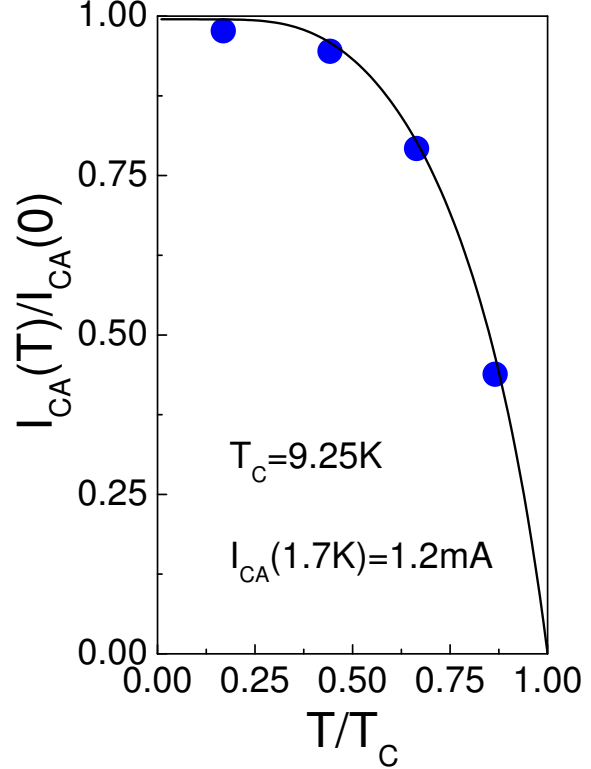


FIG. 1. (Color online) The temperature dependence of the critical current for the array of overdamped $Nb - AlO_x - Nb$ Josephson junctions. The solid line is the best fit using the single-junction Ambegaokar-Baratoff relation.

ted (solid line) to the Ambegaokar-Baratoff relation¹⁴ for the critical current of a *single junction* $I_C(T) = I_C(0) \left[\frac{\Delta(T)}{\Delta(0)}\right] \tanh \left[\frac{\Delta(T)}{2k_B T}\right]$ where $\Delta(T) = \Delta(0) \tanh \left(2.2\sqrt{\frac{T_C - T}{T}}\right)$ is the analytical approximation¹⁵ of the BCS gap parameter (valid for all temperatures) with $\Delta(0) = 1.76k_B T_C \simeq 1.5meV$ and $I_C(0) = \pi\Delta(0)/2eR \simeq 150\mu A$. This remarkable experimental fact suggests a rather strong coordinated response from all the junctions forming the array and allows us to substantially

simplify the analysis of the obtained results by considering the properties of a single junction (or plaquette, see below). For this purpose, the initial CVC data for array were rescaled by directly introducing the critical current of the single junction I_C . Some typical results of the normalized rescaled CVC taken at different temperatures are shown in Fig.2. The solid lines through the data points are the best fits according to the following expressions: $V_R = V_0 \sqrt{(I^2 - I_C^2)/I_0^2}$ for $I_C < I < I_{cr}$ and $V_C = V_0 \sqrt{(I - I_C)/I_0}$ for $I > I_{cr}$ with $V_0 = 30mV$ and $I_0 = 3mA$. It is interesting to point out that, according to Fig.2, the crossover shows almost a universal behavior taking place around $[I - I_C(T)]/I_0 \simeq 0.4$ for *all* temperatures. To understand the observed behavior of the CVC in our array, in principle one would need to analyze in detail the dynamics of the array. However, as we have previously reported^{16–20}, because of the well-defined periodic structure of our array, it is reasonable to expect that our experimental results can be quite satisfactory explained by analyzing the dynamics of a single unit cell of the array. In our calculations, the unit cell is a plaquette containing four identical Josephson junctions. By analogy with the resistively shunted junction (RSJ) model⁹, the total current in the plaquette reads¹⁶ $I = I_C(T) \sin \phi_i(t) + \frac{\Phi_0}{2\pi R} \frac{d\phi_i}{dt} + \frac{C\Phi_0}{2\pi} \frac{d^2\phi_i}{dt^2}$. Here $\phi_i(t)$ is the gauge-invariant superconducting

phase difference across the i th junction, and Φ_0 is the magnetic flux quantum. For any particular solution $\phi_i(t)$ of this equation at $I \neq 0$, the resulting CVC of the RSJ model is given by the time average of the voltage ($\tau = 2\pi/\omega$ is properly defined period) $V(I) = \frac{\hbar}{2e\tau} \int_0^\tau dt \left(\frac{d\phi_i}{dt} \right)$. For the resistance dominated situation (when the capacitance related effects can be totally neglected, that is when $\omega_{RC}\tau \gg 1$), the RSJ model has a well-known solution⁹ $\phi_i(t) = 2 \tan^{-1} \left[\frac{\hbar\omega}{2eRI} \tan\left(\frac{\omega t}{2}\right) - \frac{I}{I_C} \right]$ with $\omega = \frac{2eR}{\hbar} \sqrt{I^2 - I_C^2}$, which brings about $V_R = \frac{\hbar}{2e\tau} \int_0^\tau dt \left(\frac{d\phi_i}{dt} \right) = R\sqrt{I^2 - I_C^2}$ for R -dominated CVC. As we can see, this dependence exactly corresponds to the fitting expression for the observed CVC below I_{cr} assuming the Ohmic relation $V_0 = RI_0$. Let us turn now to the opposite situation and consider the capacitance dominated regime when the resistance related effects can be totally neglected (that is when $\omega_{RC}\tau \ll 1$). In this case, the first integral of the RSJ model reads $\frac{d\phi_i}{dt} = \sqrt{2\omega_p \sqrt{\frac{C_1}{I_C} + \frac{I}{I_C} \phi_i + \cos \phi_i}}$ where $\omega_p = \sqrt{2eI_C/\hbar C}$ is the plasmon frequency, and C_1 is the integration constant. Unfortunately, for $I \neq 0$, this equation can not be solved exactly. Hence, let us consider its approximate solution assuming that $\phi_i(t) = \frac{\pi}{2} + \theta(t)$ with $\theta(t) \ll \frac{\pi}{2}$. By fixing the arbitrary constant as $C_1 = -\frac{\pi}{2}I$, within this approximation, we obtain $\theta(t) = \Omega^2 t^2$ with $\Omega = \omega_p \sqrt{(I - I_C)/2I_C}$, which in

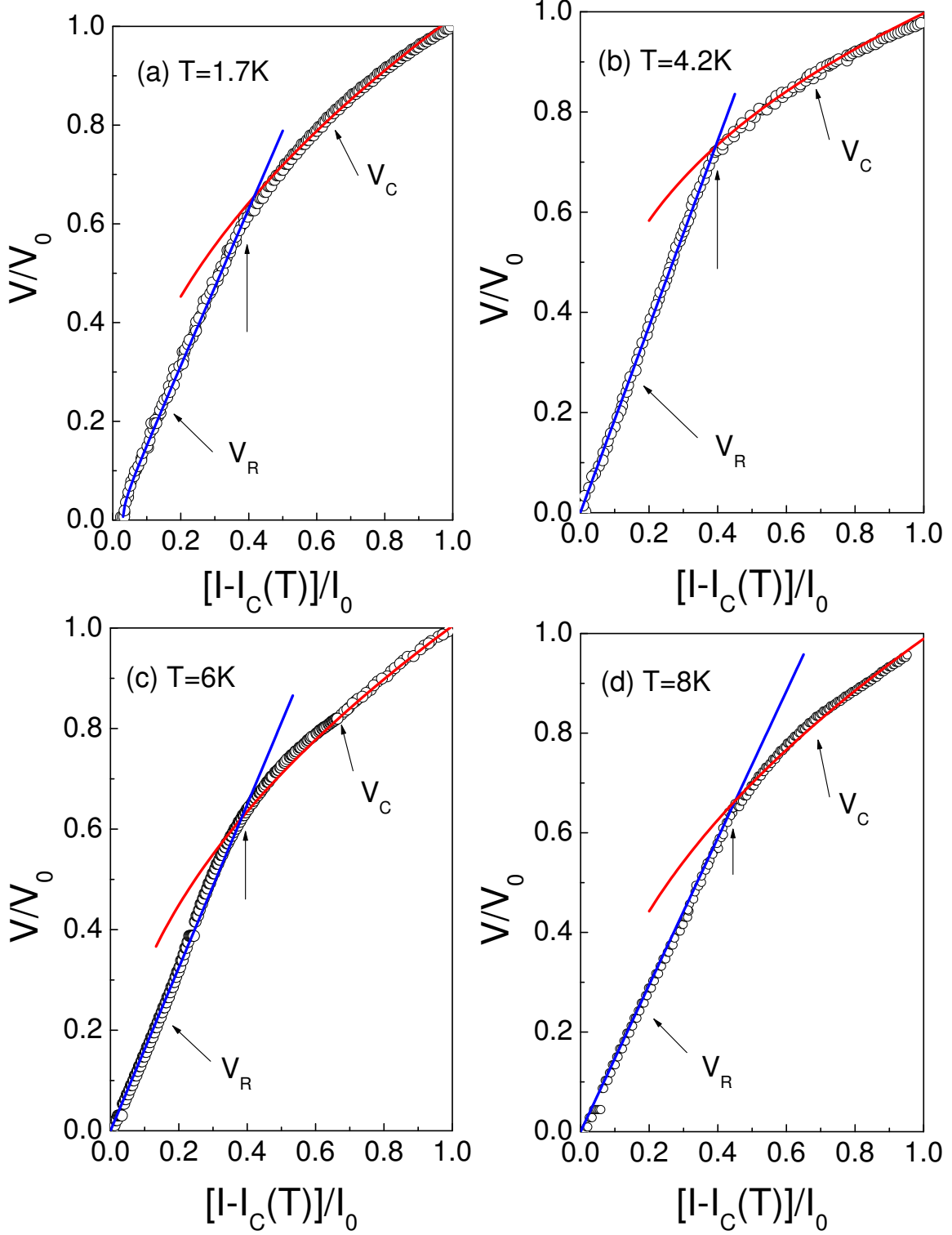


FIG. 2. (Color online) The normalized rescaled current-voltage characteristics for unshunted array of overdamped $Nb-AlO_x-Nb$ Josephson junctions taken at various temperatures along with the best fits (solid lines) using expressions for V_R and V_C (see text). Here, $I_0 = 3mA$ and $V_0 = 30mV$.

turn results in the following explicit form of C -dominated CVC, $V_C = \frac{\hbar}{2e\tau} \int_0^\tau dt \left(\frac{d\theta}{dt} \right) = \sqrt{\frac{\hbar}{4eC}} \sqrt{I - I_C}$. As we can see, this dependence exactly corresponds to the fitting expression for the observed CVC above I_{cr} assuming $V_0 = \omega_p \sqrt{2eI_0/\hbar C} = \hbar\omega_{RC}/2e$ and $I_0 = V_0/R = I_C/\beta_C$ for the normalization parameters. Furthermore, given the experimental values for R and C , we obtain $V_0 \simeq 30mV$ and $I_0 \simeq 3mA$. In turn, from the obvious identity $V_R(I_{cr}) = V_C(I_{cr})$, we readily obtain $I_{cr} = (\frac{1}{2\beta_C} - 1)I_C$ for the crossover current. Finally, a comment is in order regarding the employed here simplified model of $2D$ array. It should be noted that we completely ignored inductance (geometry) related effects which seem to be of less importance for the interpretation of the observed crossover than dissipation induced factors (resistance and capacitance). However, for more adequate description of the flux dynamics in *truly* $2D$ systems, these effects should be taken into account. Indeed, as accurate numerical simulations have revealed^{21,22}, both self-inductance and mutual inductance effects will have a significant impact on the array's dynamic properties through creation of rather strong self-induced magnetic fields.

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